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INDIAN OLYMPIAD QUALIFIER IN MATHEMATICS 2025

Note:

1. $\gcd(a, \sqrt{b})$ denotes the greatest common divisor of integers *a* and *b*.

2. For a positive real number $m, \sqrt{m}$ denotes the positive square root of *m*. For example, $\sqrt{4} = +2$

3. Unless otherwise stated all numbers are written in decimal notation.

1. If 60% of a number *x* is 40, then what is $x\%$ of 60?

Ans. **(40)**

Sol. $\frac{60}{100} \times x = 40$

$\Rightarrow x = \frac{200}{3}$

Then $x\%$ of $60 = \frac{200}{3} \times \frac{1}{100} \times 60 = 40$

2. Find the number of positive integers *n* less than or equal to 100, which are divisible by 3 but are not divisible by 2.

Ans. **(17)**

Sol. Numbers divisible by 3 less than or equal to 100 = 3, 6, 9, ………. 99

So, Total number of terms = 33 …….(1)

Numbers divisible by 2 and 3 = 6, 12,……., 96

So, Total number of terms = 16 …….(2)

Hence, Numbers divisible by 3 but not divisible by 2 = 33 – 16 = 17

3. The area of an integer-sided rectangle is 20. What is the minimum possible value of its perimeter?

Ans. **(18)**

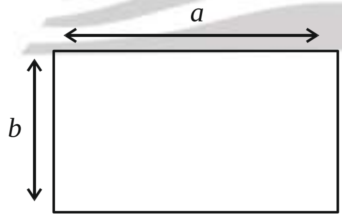
Sol. Let integer side lengths be *a* & *b*

Given $a \cdot b = 20 = 2^2 5$

Perimeter = $2(a+b)$

Possible cases of $a \cdot b = 20$ are $(1,20), (2,10), (4,5)$ with integer sides.

Least perimeter is when (4, 5) are sides i.e. $2(4+5) = 18$



4. How many isosceles integer-sided triangles are there with perimeter 23?

Ans. **(6)**

Sol. 6, 6, 11

7, 7, 9

8, 8, 7

9, 9, 5

10, 10, 3

11, 11, 1

5. How many 3-digit numbers $\overline{abc}$ in base 10 are there with $a \neq 0$ and $c = a + b$?

Ans. **(45)**

Sol. $\overline{abc} = 100a + 10b + c$

$a \in \{1, 2, \ldots, 9\}$

$b \in \{0, 1, \ldots, 9\}$

$c \in \{0, 1, \ldots, 9\}$

$a + b = c$

If $c = 9$

Number of pairs of (a, b) is 9

If $c = 8$

Number of pair of (a, b) is 8

If $c = 7$

Number of pairs of (a, b) is 7

If $c = 6$

Number of pair of (a, b) is 6

If $c = 5$

Number of pairs of (a, b) is 5

If $c = 4$

Number of pairs of (a, b) is 4

If $c = 3$

Then (a, b) pairs is 3

If $c = 2$

Then (a, b) pairs is 2

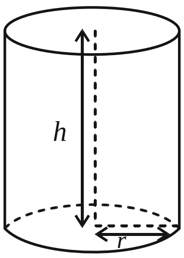
If $c = 1$

Then number of pairs (a, b) is 1.

6. The height and the base radius of a closed right circular cylinder are positive integers and its total surface area is numerically equal to its volume. If its volume is $k\pi$ where k is a positive integer, what is the smallest possible value of k?

Ans. **(54)**

Sol.



Let 'h' and 'r' be height & base radius.

Total surface area $= 2\pi r^2 + 2\pi rh$

Volume $= \pi r^2 h = k\pi \Rightarrow r^2 h = k$.

As per the question $2\pi r^2 + 2\pi rh = \pi r^2 h$

$\Rightarrow 2(r + h) = rh \Rightarrow 2\left(r + \frac{k}{r^2}\right) = \frac{k}{r}$

$\Rightarrow 2r^3 + 2k = kr \Rightarrow k = \dfrac{2r^3}{r-2}$

Since $k > 0$, $r > 2$; let $r - 2 = t \Rightarrow k = \dfrac{2(t+2)^3}{t}$ and $t = r - 2$

$\Rightarrow k = 2\left\{\dfrac{t^3 + 6t^2 + 12t + 8}{t}\right\} = 2\left\{t^2 + 6t + \dfrac{8}{t} + 12\right\}$

Now $\dfrac{t^2 + 6t + \frac{8}{t}}{15} \geq \left(t^2 \cdot t^6 \dfrac{1}{t^8}\right)^{1/15}$ (weighted $AM \geq GM$) equality holds at $t = 1$

$\Rightarrow t^2 + 6t + \dfrac{8}{t} \geq 15 \Rightarrow k \geq 2\{15 + 12\}$

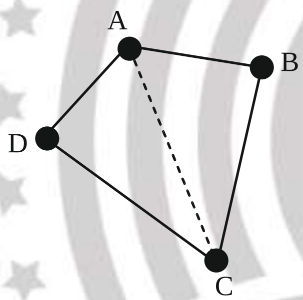
$k_{\min} = 54$ and equality holds at $r = 3$

7. A quadrilateral has four vertices *A*, *B*, *C*, *D*. We want to colour each vertex in one of the four colours red, blue, green or yellow, so that every side of the quadrilateral and the diagonal *AC* have end points of different colours. In how many ways can we do this?

Ans. **(48)**

Sol. **Case-I :** All four vertices have different colours which can be done in 4! ways.

Case-II: A and C are of different colours but Band D have the same colour other than those of $A \& C$, which can be done in $\left({}^4P_2\right) \times 2$ ways $= 24$ ways .



8. The sum of two real numbers is a positive integer *n* and the sum of their squares is $n + 1012$. Find the maximum possible value of *n*.

Ans. **(46)**

Sol. Let x and *y* are 2 real number.

Given, $x + y = n$ and $x^2 + y^2 = n + 1012$

Now, $x^2 + (n - x)^2 = n + 1012$

$\Rightarrow 2x^2 - 2nx + n^2 - n - 1012 = 0$

As $x \in R, \Delta \geq 0$

$\Rightarrow (-2n)^2 - 4 \cdot 2 \cdot \left(n^2 - n - 1012\right) \geq 0$

$\Rightarrow \quad 4n^2 - 8n^2 + 8n + 8096 \geq 0$

$\Rightarrow \quad n^2 - 2n - 2024 \leq 0$

$\Rightarrow \quad (n - 46)(n + 44) \leq 0$

$\Rightarrow \quad -44 \leq n \leq 46$

But $n \in Z^+$ $\therefore$ $1 \leq n \leq 46$

So, Maximum value of $n = 46$

9. Four sides and a diagonal of a quadrilateral are of lengths 10, 20, 28, 50, 75, not necessarily in that order. Which amongst them is the only possible length of the diagonal?

Ans. **(28)**

Sol. We use triangle inequality to identify which length can't be the length of a diagonal.

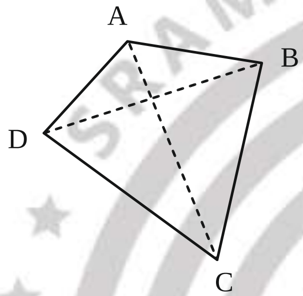
(i) 75 can't be the length of diagonal as remaining ones add up as $\{(10+20),(28+50)\}$, $\{(10+50),(28+20)\}$, $\{(10+28),(50+20)\}$ and each of them don't allow to form one or both triangles.

(ii) 50 gives us options $\{(10+20),(28+75)\},\{(10+75),(28+20)\}$, $\{(10+28),(75+20)\}$ can't be length of diagonal for same reasons.

(iii) 28 gives us options $\{(10+20),(50+75)\},\{(10+50),(28+75)\}$, $\{(10+75),(50+20)\}$ which are all okay.

Similarly, we rule out 10 and 20.

So answer is 28.



10. The age of a person (in years) in 2025 is a perfect square. His age (in years) was also a perfect square in 2012. His age (in years) will be a perfect cube *m* years after 2025. Determine the smallest value of *m*.

Ans. **(15)**

Sol. Let the age in 2025 = x^2

Let the age in 2012 = $x^2 - 13$

Given, $x^2 - 13 = y^2$ (say)

$x^2 - y^2 = 13$

$(x+y)(x-y) = 13$

$\therefore \quad \boxed{x=7, y=6}$

Age in $2025 = 7^2 = 49$

Next cube is 64, so we have to add 15.

Smallest $m = 15$

11. There are six coupons numbered 1 to 6 and six envelopes, also numbered 1 to 6. The first two coupons are placed together in any one envelope. Similarly, the third and the fourth are placed together in a different envelope, and the last two are placed together in yet another different envelope. How many ways can this be done if no coupon is placed in the envelope having the same number as the coupon?

Ans. **(40)**

Sol. **Case-I:** If $\boxed{12} \in \{3,4\}$ .... 2 options.

& $\boxed{34} \in \{5,6\}$ .... 2 options.

$\boxed{56} \in \{1,2,3\}$ or $\{1,2,4\}$ …….. 3 options.

Hence 12 ways are possible.

Case-II: If $\boxed{12} \in \{3,4\}$ ……. 2 options.

& $\boxed{34} \in \{1,2\}$ ……………… 2 options.

$\boxed{56} \in$ Remaining 2 out of $\{1,2,3,4\}$

Hence 8 ways are possible.

Case-III: If $\boxed{12} \in \{5,6\}$ ……. 2 options.

& $\boxed{34} \in \{1,2\}$ ……………… 2 options.

$\boxed{56} \in \{1,3,4\}$ or $\{2,3,4\}$ …….. 3 options.

Hence 12 ways are possible.

Case-IV: If $\boxed{12} \in \{5,6\}$ ……. 2 options.

& $\boxed{34} \in \{5,8\}$ ……………… 2 options.

$\boxed{56} \in \{1,2,3,4\}$ …….. 4 options.

Hence 8 ways are possible.

Total = 40

12. Consider five-digit positive integers of the form $\overline{abcab}$ that are divisible by the two digit number $\overline{ab}$ but not divisible by 13. What is the largest possible sum of the digits of such a number?

Ans. **(33)**

Sol. As $ab \mid \overline{ab0ab} \Rightarrow ab \mid \overline{c00}$

So we want largest possible value of $ab \,\&\, c, c = 900$.

Maximum value for $c = 9$.

We need factor of 900 with maximum $S(n)$.

$9 \times 2^2 \times 5^2$

$S(75) = 12$. By Trial and Error.

$\overline{abcab} = 75975$

13. A function *f* is defined on the set of integers such that for any two integers *m* and *n*,

$$f(mn+1) = f(m)f(n) - f(n) - m + 2$$

holds and $f(0) = 1$. Determine the largest positive integer *N* such that $\sum_{k=1}^{N} f(k) < 100$.

Ans. **(12)**

Sol. $f(mn+1) = f(m)f(n) - f(n) - m + 2$

$n = 0$

$f(1) = f(m) - 1 - m + 2$

$f(m) = m + f(1) - 1$

Put $m = 0$

$f(1) = 2$

$f(m) = m + 1 \quad \forall m \in N$ (clearly satisfies)

$\sum_{k=1}^{N} f(k) = 2 + 3 + \cdots + N + 1$

$= \frac{(N+1)(N+2)}{2} - 1 < 100$

$\therefore (N+1)(N+2) < 202$

$N \le 12$

$\max N = 12$

14. Consider a fraction $\frac{a}{b} \neq \frac{3}{4}$, where *a*, *b* are positive integers with $\gcd(a,b) = 1$ and $b \le 15$. If this fraction is chosen closest to $\frac{3}{4}$ amongst all such fractions, then what is the value of $a+b$?

Ans. **(26)**

Sol. $4a < 3b$ or $4a > 3b$

$\Rightarrow \boxed{\begin{matrix} b \le 15 \\ a \le 11 \end{matrix}}$ $\Rightarrow \boxed{\begin{matrix} b \le 15 \\ a \ge 12 \end{matrix}}$

$\because \; (a,b) = 1, \quad \because \frac{11}{15} \sim \frac{3}{4}$

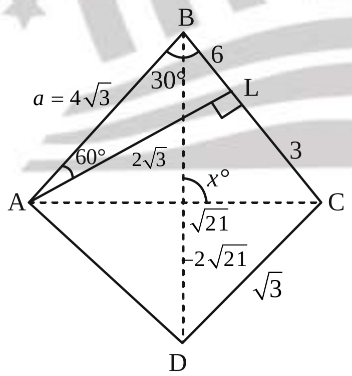
The closet fraction $\frac{a}{b} = \frac{11}{15}$

$\therefore$ Sum $a + b = 26$

15. Three sides of a quadrilateral are $a = 4\sqrt{3}, b = 9$ and $c = \sqrt{3}$. The sides *a* and *b* enclose an angle of 30°, and the sides *b* and *c* enclose an angle of 90°. If the acute angle between the diagonals is $x°$, what is the value of *x*?

Ans. **(60)**

Sol.



Area of quadrilateral ABCD = $\frac{1}{2} \times 2 \times 21 \times \sin\theta$

ABCD of quadrilateral ABCD = Area of ABL + Area of trapezium KLCD

Area of ABCD = Area of ΔABL + Area of trapezium ALCD

$\Rightarrow \frac{1}{2} \times 6(\sqrt{3})^2 + \frac{1}{2} 3\sqrt{3} \times 3 = \frac{1}{2} 21 \times 2 \sin\theta$

$\Rightarrow \frac{1}{2}\left(21\sqrt{3}\right) = \frac{1}{2} \times 21 \times 2 \sin\theta \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$

16. Let $f(x)$ and $g(x)$ be two polynomials of degree 2 such that

$$\frac{f(-2)}{g(-2)} = \frac{f(3)}{g(3)} = 4.$$

If $g(5) = 2, f(7) = 12, g(7) = -6$, what is the value of $f(5)$?

Ans. **(22)**

Sol. $\frac{f(-2)}{g(-2)} = \frac{f(3)}{g(3)} = 4$

Let $h(x) = f(x) - 4g(x)$,

$\therefore h(-2) = h(3) = 0$ & Note deg $(h) \le 2$

$h(x) = k(x+2)(x-3)$

$f(x) = 4\,g(x) + k(x+2)(x-3)$

$f(7) = 4(-6) + k(9)(4) \Rightarrow \boxed{k = 1}$

$f(5) = 4(2) + (7)(2) = \boxed{22}$

17. In triangle $ABC, \angle B = 90^\circ, AB = 1$ and $BC = 2$. On the side BC there are two points D and E such that E lies between C and D and $DEFG$ is a square, where F lies on AC and G lies on the circle through B with centre A. If the area of $DEFG$ is $\frac{m}{n}$ where m and n are positive integers with $\gcd(m, n) = 1$, what is the value of $m + n$?

Ans. **(29)**

Sol. Let the line counting GF is extended to meet AB at K and let side of square is 'a'.

$\Rightarrow AK = 1 - a$

Since ΔABC and ΔFEC are similar we get $EC = 2a$

$\Rightarrow BD = 2 - 3a = GK$

$AG^2 = AK^2 + KG^2$

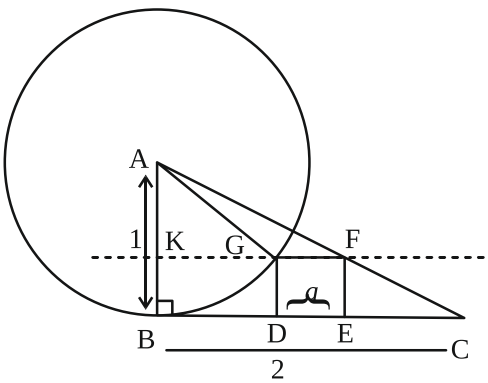
$\Rightarrow 1 = (1-a)^2 + (2-3a)^2$

$\Rightarrow 1 = 1 - 2a + a^2 + 4 - 12a + 9a^2$

$\Rightarrow 10a^2 - 14a + 4 = 0 \quad \Rightarrow 5a^2 - 7a + 2 = 0$

$\Rightarrow (5a-2)(a-1) = 0 \quad \Rightarrow a = \frac{2}{5}, 1$

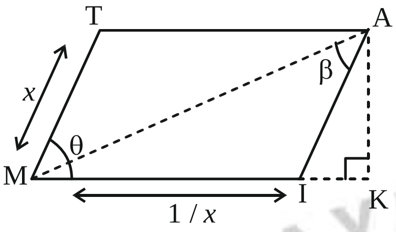
But $a \ne 1 \Rightarrow a = \frac{2}{5}$, area (DEFG) = $\frac{4}{25}$



18. MTAI is a parallelogram of area $\frac{40}{41}$ square units such that $MI = 1/MT$. If d is the least possible length of the diagonal MA, and $d^2 = \frac{a}{b}$, where a, b are positive integers with $\gcd(a,b)=1$, find $|a-b|$.

Ans. **(23)**

Sol.



Let the perpendicular from A on extended MI meets at K.

ar $(MTAI) = MI.AK = \frac{40}{41}$ …….(1)

Also $MT.MI = 1 = MI.AI$ …….(2)

ar $(AMTI) = MI.MT\sin\theta \Rightarrow \sin\theta = \frac{40}{41}$

$MA = \sqrt{x^2 + \frac{1}{x^2} + 2\cos\theta}$ (using cosine rule)

$\Rightarrow MA_{\min} = \sqrt{2(1+\cos\theta)}$ (when $x=1$)

$= \sqrt{2\left(1-\frac{9}{41}\right)} = \sqrt{\frac{64}{41}} \Rightarrow d^2 = \frac{64}{41} \Rightarrow |a-b| = 23$

19. Let N be the number of nine-digit integers that can be obtained by permuting the digits of 223334444 and which have at least one 3 to the right of the right-most occurrence of 4. What is the remainder when N is divided by 100?

Ans. **(40)**

Sol. Consider the position of rightmost 3 among all three 3's.

Clearly there can't be any 4 on right of this last 3. So there can be only 2's after this 3.

C_1 : The right most 3 in Unit's place

Remaining 22, 33, 4444 can be permuted in $\frac{8!}{4!2!2!} = \frac{8.7.6.5}{2.2} = 420$ ways

C_2 : The rightmost 3 in Ten's place so no. ends with 32.

Remaining 2, 33, 4444 can be permuted in $\frac{7!}{4!2!} = \frac{7.6.5}{2} = 105$ ways

C_3 : The rightmost 3 in hundred's place so no. ends with 322

Remaining 33,4444 can be permuted in $\binom{6}{2} = 15$ ways

$\therefore$ Total number of good permutations $(\text{mod}100) = \boxed{40}$

20. Let f be the function defined by

$$f(n) = \text{remainder when } n^n \text{ is divided by } 7,$$

for all positive integers n. Find the smallest positive integer T such that $f(n+T) = f(n)$ for all positive integers n.

Ans. **(42)**

Sol. $f(n) = n^n (\bmod 7)$

$(n+T)^{(n+T)} = n^n (\bmod 7)$

For $n \equiv 0(\bmod 7)$, $(n+T)^{(n+T)} \equiv 0(\bmod 7)$.

$\Rightarrow T \equiv 0(\bmod 7)$.

So for all $n \in N$, $(n+T)^{n+T} \equiv n^{n+T} \equiv n^n \cdot n^T \equiv n^T \equiv 0(\bmod 7)$

As order $7^{(n)} | \phi(7) = 6$.

$\therefore 6 | T$

Hence smallest such $T = 6 \times 7 = 42$.

21. Let $P(x) = x^{2025}, Q(x) = x^4 + x^3 + 2x^2 + x + 1$. Let $R(x)$ be the polynomial remainder when the polynomial $P(x)$ is divided by the polynomial $Q(x)$. Find $R(3)$.

Ans. **(53)**

Sol. $x^{2025} = \left(x^4 + x^3 + 2x^2 + x + 1\right)T(x) + R(x)$ ....(1)

$R(x) = ax^3 + bx^2 + cx + d$

Roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$ is $i, -i, \omega, \omega^2$

Put $x = i$ in (1) we get $b = d$ and $c = a + 1$

Put ω in (1) we get $b = 2$, $c = 2$, $d = 2$, $a = 1$

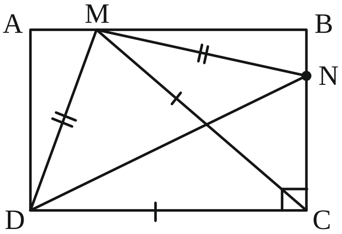
$\Rightarrow R(x) = x^3 + 2x^2 + 2x + 2$

$\Rightarrow R(3) = 53$

22. Let $ABCD$ be a rectangle and let M, N be points lying on sides AB and BC, respectively. Assume that $MC = CD$ and $MD = MN$, and that points C, D, M, N lie on a circle. If $(AB/BC)^2 = m/n$ where m and n are positive integers with $\gcd(m,n) = 1$, what is the value of $m+n$?

Ans. **(3)**

Sol.



MNCD ⊙ $\angle NCD = 90° \Rightarrow \angle DMN = 90°$

$\Rightarrow \angle MDN = 45°$

$\Rightarrow \angle MCN = 45°$

$\Rightarrow \angle MCB = 45°$

$\Delta MCB \Rightarrow \dfrac{MC}{BC} = \sqrt{2}\left(= \dfrac{1}{\cos 45°}\right)$

$MC = CD = AB$

$\frac{AB}{BC} = \sqrt{2} \quad \Rightarrow \left(\frac{AB}{BC}\right)^2 = 2 = \frac{m}{n} \Rightarrow m + n = \boxed{3}$

23. For how many numbers n in the set $\{1, 2, 3, \ldots, 37\}$ can we split the $2n$ numbers $1, 2, \ldots, 2n$ into n pairs $\{a_i, b_i\}, 1 \le i \le n$, such that $\prod_{i=1}^{n}(a_i + b_i)$ is a square?

Ans. **(36)**

Sol. For n even then split

$1, 2, ...2n$ as $(1, 2n), (2, 2n-1).........(n-1, n+1)$

So, required product $= (2n+1)^n$ = Perfect square as n is even.

For odd n, consider pairs $(1,5),(2,4),(3,6) \rightarrow$ Product $= 6^2 \cdot 9 = 18^2$

Now remaining nos. can be pared up as $(7, 2n)(8, 2n-1), \cdots (n+3, n+4)$ giving product of each pair $(2n+7)$ & there are $(n - 3)$ pairs.

Product $= 18^2 (2n+7)^{n-3}$ when $n - 3$ is even.

Hence P is perfect square.

Hence it's possible for all $n \ge 2$.

24. There are m blue marbles and n red marbles on a table. Arman and Babita play a game by taking turns. In each turn the player has to pick a marble of the colour of his/her choice. Arman starts first, and the player who picks the last red marble wins. For how many choices of (m, n) with $1 \le m, n \le 11$ can Arman force a win?

Ans. **(66)**

Sol. Let $f(R,B) = \begin{cases} 1, \text{ if the first player has a winning strategy.} \\ -1, \text{ otherwise} \end{cases}$

Clearly, $f(R, B) = -f(R-1, B) = f(R, B-1)$

Also, $f(1, B) = 1 \qquad \forall B \in N$ and

$f(R,1) = \begin{cases} 1 & \text{,R-odd.} \\ -1 & \text{,R-even.} \end{cases}$

Claim: For $R > 1$

$f(R,B) = \begin{cases} 1 & \text{if } R + B = \text{ odd} \\ -1 & \text{if } R + B = \text{ even} \end{cases}$

Proof : We will prove this by induction on (R + B)

Base case : (1) $f(2,1) = -f(2,0) = +f(1,0) = 1$

(2) $f(2,2) = -f(2,1) = +f(2,0) = -f(1,0) = -1$

Let $f(R,B) = \begin{cases} 1 & \text{if } R + B = \text{ odd} \\ -1 & \text{if } R + B = \text{ even} \end{cases}$

for all values of $R + B \le n$

For $R + B = n + 1$

Consider $f(R+1, B) = -f(R, B)$

$\Rightarrow f(R+1,B)=\begin{cases}-1 & \text{if } R+B= \text{ odd}\\ 1 & \text{if } R+B= \text{ Even}\end{cases}$

$\Rightarrow \ f(R+1,B)=\begin{cases}-1 & \text{if } R+B+1= \text{ Even}\\ 1 & \text{if } R+B+1= \text{ odd}\end{cases}$

Hence there will be 11 solution $f(1,B)$ and out of remaining 110 we get exactly 55 solutions.

$\therefore$ Totally 66 solutions.

25. For some real numbers m, n and a positive integer a, the list $(a+1)n^2, m^2, a(n+1)^2$ consists of three consecutive integers written in increasing order. What is the largest possible value of m^2?

Ans. **(49)**

Sol. $a(n+1)^2-(a+1)n^2=2$

$a\left(n^2+2n+1\right)-(a+1)n^2=2.$

$an^2+2an+a-an^2+n^2=2$

$n^2-2an=a-2.$

$n^2-2an+a^2=a^2+a-2.$

$(n-a)^2=(a+2)(a-1)=$ perfect square

Let $n-a=k$

$\therefore \quad k^2=(a+2)(a-1)$

$4k^2=4a^2+4a-8.$

$(2a+1)^2-(2k)^2=9.$

C_1 : $2a-2k+1=3$ & $2a+2k+1=3$

$\Rightarrow a=1, \quad k=0$ so, $\boxed{n=a}$

No. become 2, 3, 4 but $m^2=3$ not a perfect square

C_2: $2a-2k+1=1$ & $2a+2k+1=9$

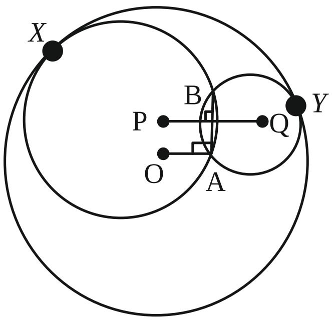
$a=k=2, \quad n=4.$

No. are 48, 49, 50.

26. Let S be a circle of radius 10 with centre O. Suppose S_1 and S_2 are two circles which touch S internally and intersect each other at two distinct points A and B. If $\angle OAB=90^\circ$ what is the sum of the radii of S_1 and S_2?

Ans. **(10)**

Sol.



Centres P, Q

Radii of $\odot$ $O, P, Q = R, r_1, r_2$ respectively

R = 10

$OP+PA=OP+PX=OX=R$

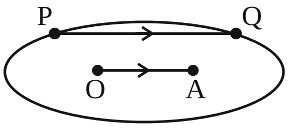
$OQ+QA=OQ+QY=OY=R$

$\therefore$ P, Q lie on ellipse with focii O, A and major axis $OP+PA=R$ (Fixed ellipse).

But $\angle OAB=90°$ & $PQ\perp AB$

$\therefore$ PQ || OA

PQ || Major axis.



$\therefore$ P, Q symmetric w.r.t minor axis.

$\therefore$ $OP=QA \Rightarrow R-r_1=r_2$

$\therefore$ $r_1+r_2=R=\boxed{10}$

27. A regular polygon with $n\geq 5$ vertices is said to be colourful if it is possible to colour the vertices using at most 6 colours such that each vertex is coloured with exactly one colour, and such that any 5 consecutive vertices have different colours. Find the largest number n for which a regular polygon with n vertices is not colourful.

Ans. **(19)**

Sol. Notice that by virtue of every 5 consecutive vertices being of different colours and us having only at most 6 colours, we can pick any 5 colours for the first 5 vertices and there after we have just 2 choices for the 6th vertex (either same as first or the unused colour). Therefore, if we create 5-colour and 6-colour blocks we can use them freely to colour our regular polygon. Thus, any n of the form $5x+6y$ can be colured. This means all numbers above 19 (chicken mcnugget theorem) are certainly colourable. At 19 through, no combination of 5 and 6 colour blocks will work, so our answer is 19.

28. Find the number of ordered triples (a,b,c) of positive integers such that $1\leq a,b,c\leq 50$ which satisfy the relation

$$\frac{\text{lcm}(a,c)+\text{lcm}(b,c)}{a+b}=\frac{26c}{27}.$$

Here, by $\text{lcm}(x,y)$ we mean the LCM, that is, least common multiple of x and y.

Ans. **(40)**

Sol. $a=gprx$

$b=gpqy$

$c=gqrz$

$L(a,c)=gpqrxz$

$L(b,c)=gpqryz$

Given equation $\dfrac{gpqrz(x+4)}{gp(rx+qy)}=\dfrac{26\cdot gqrz}{27}$

$\dfrac{(x+y)}{(rx+qy)}=\dfrac{26g}{27}\leq 1$

$\Rightarrow \boxed{g=1}$

If r, q are both ≥ 2, then LHS $\leq \frac{1}{2}$, Not possible so area of them is 1.

Case-I : $\boxed{r=1}$

$27(x+y) = 26\,x + 2694$

$x + 27y = 26qy$

$x = (26q - 27)y$

As $(x, y) = 1 \Rightarrow\ y = 1,\ \ x = 26q - 27\ < 50$

$\Rightarrow \boxed{q=2}$ is only possibility & $\boxed{x=25}$

$\boxed{n=25}$ $\boxed{y=1}$ $\boxed{q=2}$ $\boxed{r=1}$ $\boxed{g=1}$

$a = p25 \ \ \Rightarrow\ \ p = 1\ or\ 2$

$b = p21$

$c = 2z \ \ \Rightarrow \ \ z \in \{1, 2 \ldots 25\}$ but $g(x, z) = 1$

So, z Takes 20 values.

Note $p \neq 2$ or $g = 2$

We get 25 triples.

Case-II : $q = 1 \quad r = 2$

a, b will interchange.

$\therefore$ Total no of triples = 40

29. Consider a sequence of real numbers of finite length. Consecutive four term averages of this sequence are strictly increasing, but consecutive seven term averages are strictly decreasing. What is the maximum possible length of such a sequence?

Ans. **(10)**

Sol. Let the sequence have 11 terms if possible. Then by 4-average condition, we get $a_1 < a_5$, $a_2 < a_6$...., $a_7 < a_{11}$. But this implies the 7-term average from a_1 is larger than that from a_5 which is a contradiction, thus the answer is 10. To construct a valid sequence consider this: 3, 6, 9, 1, 4, 7, 10, 2, 5, 8. Please check if this sequence satisfied the conditions.

30. Assume a is a positive integer which is not a perfect square. Let x, y be non-negative integers such that $\sqrt{x - \sqrt{x+a}} = \sqrt{a} - y$. What is the largest possible value of a such that $a < 100$?

Ans. **(91)**

Sol. C_1 : Let $y \neq 0$. Squaring we get

$x - \sqrt{x+a} = a + y^2 - 2y\sqrt{a}.$

Note $a \in Z, a$ is not perfect square $\Rightarrow \sqrt{a} \notin Q$

Equating rational and irrational parts. We get

$x = y^2 + a \qquad \& \qquad x + a = 4y^2 a$

$y^2 + 2a = 4y^2 a$

$\Rightarrow 4y^2 a - y^2 - 2a = 0$

$8y^2 a - 2y^2 - 4a = 0$

$(4a-1)\left(2y^2 - 1\right) = 1$

$\Rightarrow \boxed{4a=2}\times$ or $2y^2=2,\ y=1$

$x=a+1,\ x+a=4a,\ \ x=3a=a+1$: Not possible

C_2 : Hence $y=0$

$x-\sqrt{x+a}=a$

$\therefore\ \ (x-a)^2=x+a$

$\therefore\ \ x^2-2ax+a^2=x+a$

$x^2-(2a+1)x+\left(a^2-a\right)=0$, has integer solution

$\therefore\ \Delta=(2a+1)^2-4\left(a^2-a\right)=k^2$ for some k

$=4a^2+4a+1-4a^2+4a=8a+1=k^2$

As a $<100,\ k^2<801$

For maximum $k=729$, gives

$\boxed{a=91}$